

MAT 534: HOMEWORK 7
DUE THU OCT 14

Problems marked by asterisk (*) are optional.

1. Dummit and Foote, problems 11, 13 and 14 on p. 231.
2. Let $p \in \mathbb{R}[x]$ be a quadratic polynomial which has no real roots. Define $R = \mathbb{R}[x]/(p)$. Show that $R \cong \mathbb{C}$.
3. Let $I = (x - y)$, $J = (x + y)$ be ideals in $\mathbb{C}[x, y]$.
 - (a) Describe explicitly the rings
 $\mathbb{C}[x, y]/I$, $\mathbb{C}[x, y]/J$, $\mathbb{C}[x, y]/(I + J)$, $\mathbb{C}[x, y]/IJ$.
(Hint: you may make change of variables $x' = x + y$, $y' = x - y$).
Describe each of these rings as polynomial functions on a certain subset in $\mathbb{C}^2$.
 - (b) Which of the ideals I , J , $I + J$, IJ is maximal? prime?
4. Dummit and Foote, problems 7 and 8 on p. 256.
- *5. Dummit and Foote, problems 33 and 34 on p. 259.